

This is the second of our "spiraling review" homework sets.

Write your homework *neatly, in pencil*, on $8\frac{1}{2} \times 11$ blank white printer paper (the back can be used). Always *write the problem*, or at least enough of it so that your work is readable. In particular, you *must* write any function the problem refers to.

Learn to *write in sentences*. Use words, sentences, paragraphs when appropriate. Sentences begin with a word and end with a period. Avoid having apparently random expressions and equations scattered around the page. Justify your conclusions.

Problem 1. Let

$$f(x) = \frac{x}{1+x^2}.$$

Find the slope of the line tangent to the graph of f at the point $(2, \frac{2}{5})$.

Problem 2 (Thomas §4.1 # 59). The function

$$V(x) = x(10 - 2x)(16 - 2x) \quad \text{for} \quad 0 < x < 5$$

models the volume of a box.

(a) Find the extreme values of V .

(b) Interpret any values found in part (a) in terms of volume of the box.

Problem 3 (Thomas §4.1 # 66). If an even function $f(x)$ has a local maximum at $x = c > 0$, can anything be said about the value of f at $x = -c$? Justify your answer.

Problem 4 (Thomas §4.1 # 67). If an odd function $g(x)$ has a local maximum at $x = c > 0$, can anything be said about the value of g at $x = -c$? Justify your answer.

Problem 5 (Thomas §4.1 # 69). Consider a generic cubic function

$$f(x) = ax^3 + bx^2 + cx + d.$$

(a) Show that f can have 0, 1, or 2 critical points. Give examples and graphs to support your argument.

(b) How many local extreme values can f have?

Problem 6. Compute

$$\int_0^1 x^2 \tan(x^3) dx.$$

Problem 7 (Thomas §3.6 # 30). Consider the equation

$$x + \sin y = xy.$$

Use implicit differentiation to find dy/dx .

Problem 8 (Re: Thomas §3.6 # 30). Consider the equation

$$y + \sin x = xy.$$

- (a) Solve for y so that y is a function of x . Let $f(x) = y$.
- (b) Graph your function on a graphing calculator, and sketch the graph.
- (c) What is the domain of f ?
- (d) Where does the equation $y + \sin x = xy$ implicitly define y as a function of x ?
- (e) Where does the equation $x + \sin y = xy$ implicitly define x as a function of y ?

Problem 9. Compute

$$\lim_{h \rightarrow 0} \frac{\sin(a+h) - \sin a}{h},$$

where $a = \pi/3$.

Problem 10. Let

$$f(x) = x^4 - 32x.$$

Find the range of f .